On the Nonrelativistic Limit of the Scattering of Spin One-half Particles Interacting with a Chern–Simons Field

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Abstract

Starting from a relativistic quantum field theory, we study the low energy scattering of two fermions of opposite spins interacting through a Chern-Simons field. Using the Coulomb gauge we implement the one loop renormalization program and discuss vacuum polarization and magnetic moment effects. We prove that the induced magnetic moments for spin up and spin down fermions are the same. Next, using an intermediary auxiliary cutoff the scattering amplitude is computed up to one loop. Similarly to Aharonov-Bohm effect for spin zero particles, the low energy part of the amplitude contains a logarithmic divergence in the limit of very high intermediary cutoff. In our approach however the needed counterterm is automatically provided without any additional hypothesis.

I. INTRODUCTION

Recent studies have unveiled interesting aspects of the scattering of charged particles by a thin magnetic flux tube, the Aharonov-Bohm (AB) effect [1]. Such investigations were motivated by some conceptual difficulties in the case of spin zero particles [2]. In that situation, it has been shown that accordance between the exact and perturbative calculations can be achieved only after the inclusion in the perturbative method of a contact, delta

like interaction. As remarked in [3], the discrepancy between the two results, prior to the addition of the delta interaction, was to be expected since different boundary conditions were adopted. In a field theory context, taking the AB and anyon scatterings in (2+1) dimensions as equivalent, the contact interaction may be simulated by a $(\phi^*\phi)^2$, where ϕ is the particle's field. This procedure was employed to reproduce the AB scattering in a direct nonrelativistic approach [3] and also as the low energy limit of the relativistic theory of a scalar field minimally coupled to a Chern–Simons field [4].

It has been argued that in the fermionic case the additional auto–interaction is not needed since it is automatically provided by Pauli's magnetic term. These issues have been recently examined [5, 6] from two different perspectives. In [5] we examined the scattering of two low energy spin up fermions, starting from a relativistic quantum field formulation and in [6] the same amplitude was calculated through a Galilean formulation of field theory as described by Lévy–Leblond [7]. As expected, the amplitudes obtained by both methods were free from ultraviolet divergences, in accord with the above mentioned conjecture for the scattering of two spin up fermions. Nevertheless, our results show differences with respect to the nonrelativistic treatment coming from vacuum polarization, induced magnetic moment and also from the exchange of two "photons". In reference [6], on the other hand, it was also considered the scattering of a spin up and a spin down fermions, with the conclusion that the Pauli magnetic interaction cancels and a logarithmic divergence shows up. Motivated by these developments we here extend our study of [5] to the scattering of a spin up and a spin down fermions starting from a relativistic quantum field formulation. By introducing an intermediary cutoff, we separate the regions of low and high virtual momenta in the Feynman integrals. This allows a direct simplification of the integrands and suggests the identification of the low momentum part with the corresponding Galilean amplitude got in [6]. In our approach, however, the logarithmic divergence is canceled by a contribution coming from the complementary high momentum part, leaving us with a finite result. We also determine an effective Lagrangian which summarizes the low momentum behavior of a system of two relativistic fermions interacting through a CS field.

II. TREE APPROXIMATION AND ONE LOOP RENORMALIZATION

The Lagrangian of the system is

$$\mathcal{L} = \frac{\theta}{4}\epsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha + \bar{\psi}(i\not{\partial} - m)\psi + \bar{\varphi}(i\not{\partial} + m)\varphi + e\bar{\psi}\gamma^\mu\psi A_\mu + e\bar{\varphi}\gamma^\mu\varphi A_\mu, \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and ψ (φ) is a two component Dirac field representing particles and anti-particles of spin up (down) and the same mass m (the parameter m is to be taken positive). We shall be interested in getting the low energy approximation for the scattering amplitude of one particle spin up by another of spin down (total spin zero). For completeness, we shall also reproduce the corresponding results for the case of two spin up fermions but before anti-symmetrization, as done in [5]. Assuming that in the center of mass frame the incoming and outgoing particles have momenta $p_1 = (w_p, \vec{p})$, $p_2 = (w_p, -\vec{p})$ and $p_1{}' = (w_p, \vec{p}\,')$, $p_2{}' = (w_p, -\vec{p}\,')$, where $|\vec{p}| = |\vec{p}\,'|$ and $w_p = \sqrt{m^2 + \vec{p}^2}$, the tree approximation for these processes are respectively [8],

$$T^{(0)}_{\uparrow\downarrow} = -ie^2\bar{v}(\vec{p}\,')\gamma^\mu v(\vec{p})\Delta_{\mu\nu}(\vec{p}\,' - \vec{p})\bar{u}(-\vec{p}\,')\gamma^\nu u(-\vec{p}) \tag{2.2}$$

$$T^{(0)}_{\uparrow\uparrow} = -ie^2\bar{u}(\vec{p}\,')\gamma^\mu u(\vec{p})\Delta_{\mu\nu}(\vec{p}\,' - \vec{p})\bar{u}(-\vec{p}\,')\gamma^\nu u(-\vec{p}) \tag{2.3}$$

where the CS field propagator in the Coulomb gauge [9] is given by

$$\Delta_{\mu\nu}(k) = \frac{1}{\theta}\epsilon_{\mu\nu\rho}\frac{\bar{k}^\rho}{\vec{k}^2}, \tag{2.4}$$

with $\bar{k}^\alpha \equiv (0, \vec{k})$.

After expanding in powers of $\frac{|\vec{p}|}{m}$ ($\ll 1$), we get in leading order,

$$T^{(0)}_{\uparrow\downarrow} = \frac{ie^2}{\theta m}\frac{\vec{s}\wedge\vec{q}}{\vec{q}^2} \tag{2.5}$$

$$T^{(0)}_{\uparrow\uparrow} = \frac{e^2}{\theta m}(1 + i\frac{\vec{s}\wedge\vec{q}}{\vec{q}^2}) \tag{2.6}$$

where $\vec{s} = \vec{p} + \vec{p}\,'$, $\vec{q} = \vec{p}\,' - \vec{p}$ and $\vec{s}\times\vec{q}$ stands for $\epsilon^{ij}s_i q_j$. As can be seen in a direct nonrelativistic treatment [5, 6], the origin of the constant term in Eq. (2.6) is a contact Pauli interaction between the magnetic moment of each fermion with the magnetic field of

the other. In the anti-parallel case, Eq. (2.5), these effects cancel each other and this is the basic reason behind the above mentioned discrepancy for the spin zero scalar case.

Before embarking into the discussion of the one loop scattering amplitudes, let us comment on the renormalization of the model (2.1). As in the case of just one fermion field interacting with the CS field, [10], the divergences are concentrated into contributions to the vacuum polarization, self energies and vertex parts. The analysis is entirely analogous to that of Ref. [10]. Now, however,there are two contributions to the CS self energy,

$$\Pi^{\alpha\beta}(k) = -e^2 \int \frac{d^3q}{(2\pi)^3} \frac{\mathrm{Tr}[\gamma^\alpha(\not{q}+\not{k}+m)\gamma^\beta(\not{q}+m)]}{[(k+q)^2-m^2](q^2-m^2)} + (m \to -m), \tag{2.7}$$

and the would be induced CS term vanishes. Therefore θ is kept unchanged and can be considered as the renormalized CS parameter. For low momentum we get

$$\Pi^{\alpha\beta}(k) = -i\frac{e^2}{6\pi m}(k^2 g^{\alpha\beta} - k^\alpha k^\beta), \tag{2.8}$$

implying that the effective low energy Lagrangian will contain a Maxwell term with intensity twice as big as in the case of just one fermion field.

Defining the ψ field self-energy, Σ_ψ from the propagator $S_{\psi F}(p) = i(\not{p} - m + i\Sigma_\psi)^{-1}$, we have

$$\Sigma_\psi(p) = -\frac{ie^2}{\theta}\int \frac{d^3k}{(2\pi)^3} \frac{\gamma^\mu(\not{k}+\not{p}+m)\gamma^\nu}{(k+p)^2-m^2} \frac{\epsilon_{\mu\nu\rho}\bar{k}^\rho}{(\vec{k})^2}\,. \tag{2.9}$$

Choosing a mass counterterm so that m is the physical mass, i. e., the position of the propagator's pole, we obtain the renormalized result

$$\Sigma_\psi = -\frac{ie^2}{2\pi\theta}\Big[\frac{(\vec{p}^2 - m\vec{p}\cdot\vec{\gamma})}{m+w_p}\Big]. \tag{2.10}$$

The self–energy of the φ field is very similar, giving $(S_{\varphi F}(p) = i(\not{p} + m + i\Sigma_\varphi)^{-1})$

$$\Sigma_\varphi = -\frac{ie^2}{2\pi\theta}\Big[\frac{(\vec{p}^2 + m\vec{p}\cdot\vec{\gamma})}{m+w_p}\Big]. \tag{2.11}$$

There are two vertex parts to be considered. The calculation of these vertex parts is greatly simplified if it is restricted to the low momentum region. For the φ field up to one loop we have,

$$\Gamma^\rho_\varphi(p,p') = \frac{ie^3}{\theta}\int \frac{d^3k}{(2\pi)^3}\frac{\gamma^\mu(\not p' - \not k - m)\gamma^\rho(\not p - \not k - m)\gamma^\alpha \epsilon_{\alpha\mu\nu}\bar{k}^\nu}{[(p'-k)^2 - m^2 + i\epsilon][(p-k)^2 - m^2 + i\epsilon](-\vec{k}^2)} + ie\gamma^\rho \tag{2.12}$$

so that, in the low momentum regime,

$$\bar{v}(p')\Gamma^0_\varphi v(p) = ie, \tag{2.13}$$

$$\bar{v}(p')\Gamma^i_\varphi v(p) = ie\left(1+\frac{e^2}{4\pi\theta}\right)\frac{(p+p'^i)}{2m} - e\left(1-\frac{e^2}{4\pi\theta}\right)\epsilon^{ij}\frac{(p'-p)^j}{2m}. \tag{2.14}$$

The vertex part of the ψ field is similar and one only has to observe the changes due to the different sign of the mass (see Ref. [5])

Coupling, alternatively, an external electromagnetic field to the fields φ and ψ, one could compute quantum corrections to their respective magnetic moments, Fig. 1. We would like to emphasize that, contrarily to what happens in covariant gauges, in the Coulomb gauge it is essential to take into account self energy corrections to obtain the correct result. In particular, for spin down fermions scattered by an effective external field $\mathcal{A}^\mu(q)$, which already incorporates the polarization of the vacuum, we obtain the scattering amplitude

$$ie\mathcal{A}^0(q) - i\frac{e}{2m}s^i\mathcal{A}^i(q) + \frac{e}{2m}(1-\frac{e^2}{2\pi\theta})\epsilon^{ij}q^j\mathcal{A}^i(q). \tag{2.15}$$

From this expression, one sees that the sole effect of the vertex correction is to modify the fermion magnetic moment which now turns out to be

$$\mu_{down} = -\frac{e}{2m}\Big(1-\frac{e^2}{4\pi\theta}\Big). \tag{2.16}$$

This result must be compared with

$$\mu_{up} = \frac{e}{2m}\Big(1+\frac{e^2}{4\pi\theta}\Big) \tag{2.17}$$

got in [5] for the spin up field ψ. It should be noticed that the induced magnetic moment has the same sign in both cases. This is in accord with the results for the anomalous spin [6, 11, 12]. Therefore, the total intensity of the magnetic moment increases for one field and lowers for the other.

III. ONE LOOP SCATTERING

We are now ready to pursue our analysis of the scattering of spin 1/2 particles at one loop order. First of all, self–energy and radiative corrections to the tree approximation, in leading $1/m$ order, give

$$T_{\uparrow\downarrow R} = \frac{e^4}{6\pi m\theta^2} + \frac{e^4}{2\pi m\theta^2} = \frac{2e^4}{3\pi m\theta^2}, \tag{3.1}$$

where the first and second terms in the first equality come, respectively, from the vacuum polarization and vertex insertions. It is easy to verify that the same expression holds for the spin-up/spin-up case (the difference of this result with respect to the one in [5] is just because we now have two fields instead of one contributing to the vacuum polarization tensor).

The remaining graphs to be analyzed are shown in Fig. 2. The box and crisscross two photon exchange amplitudes, for the total spin zero case, are given by

$$\begin{aligned} T_{\uparrow\downarrow B} = ie^4 \int \frac{d^3k}{(2\pi)^3} &\Big[\bar{v}(\vec{p}\,')\gamma^\mu S_F(r,-m)\gamma^\nu v(\vec{p})\Big] \\ &\Big[\bar{u}(-\vec{p}\,')\gamma^\alpha S_F(r',m)\gamma^\beta u(-\vec{p})\Big]\Delta_{\nu\beta}(\vec{k}-\vec{p})\Delta_{\alpha\mu}(\vec{k}-\vec{p}\,') \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} T_{\uparrow\downarrow X} = ie^4 \int \frac{d^3k}{(2\pi)^3} &\Big[\bar{v}(\vec{p}\,')\gamma^\mu S_F(r,-m)\gamma^\nu v(\vec{p})\Big] \\ &\Big[\bar{u}(-\vec{p}\,')\gamma^\alpha S_F(t,m)\gamma^\beta u(-\vec{p})\Big]\Delta_{\nu\alpha}(\vec{k}-\vec{p})\Delta_{\beta\mu}(\vec{k}-\vec{p}\,'). \end{aligned} \tag{3.3}$$

where $r \equiv (w_p + k^0, \vec{k})$, $r' \equiv (w_p - k^0, -\vec{k})$ and $t \equiv (w_p + k^0, \vec{k} - \vec{p} - \vec{p}\,')$.

To simplify the calculation we shall use

$$S_F(p,-m) = i\frac{\not{p} - m}{p^2 - m^2 + i\epsilon} = i\frac{v(\vec{p})\bar{v}(\vec{p})}{p^0 - w_p + i\epsilon} + i\frac{u(-\vec{p})\bar{u}(-\vec{p})}{p^0 + w_p - i\epsilon}. \tag{3.4}$$

for the φ free field propagator. The propagator of the free ψ field, $S_F(p,m)$, is obtained from (3.4) replacing m by $-m$ and exchanging the spinors u and v. Using these expressions we get,

$$
\begin{aligned}
T_{\uparrow\downarrow B} &= -ie^4 \int \frac{d^3k}{(2\pi)^3} \Delta_{\nu\beta}(\vec{k}-\vec{p})\Delta_{\alpha\mu}(\vec{k}-\vec{p}\ ')\\
&\bar{v}(\vec{p}\ ')\gamma^\mu \Big[\frac{v(\vec{k})\bar{v}(\vec{k})}{k^0+w_p-w_k+i\epsilon} + \frac{u(-\vec{k})\bar{u}(-\vec{k})}{k^0+w_p+w_k-i\epsilon}\Big]\gamma^\nu v(\vec{p})\\
&\bar{u}(-\vec{p}\ ')\gamma^\alpha \Big[\frac{u(-\vec{k})\bar{u}(-\vec{k})}{w_p-k^0-w_k+i\epsilon} + \frac{v(\vec{k})\bar{v}(\vec{k})}{w_p-k^0+w_k-i\epsilon}\Big]\gamma^\beta u(-\vec{p})
\end{aligned} \tag{3.5}
$$

and

$$
\begin{aligned}
T_{\uparrow\downarrow X} &= -ie^4 \int \frac{d^3k}{(2\pi)^3} \Delta_{\nu\alpha}(\vec{k}-\vec{p})\Delta_{\beta\mu}(\vec{k}-\vec{p}\ ')\\
&\bar{v}(\vec{p}\ ')\gamma^\mu \Big[\frac{v(\vec{k})\bar{v}(\vec{k})}{k^0+w_p-w_k+i\epsilon} + \frac{u(-\vec{k})\bar{u}(-\vec{k})}{k^0+w_p+w_k-i\epsilon}\Big]\gamma^\nu v(\vec{p})\\
&\bar{u}(-\vec{p}\ ')\gamma^\alpha \Big[\frac{u(\vec{k}-\vec{s})\bar{u}(\vec{k}-s)}{k^0+w_p-w_{k-s}+i\epsilon} + \frac{v(\vec{s}-\vec{k})\bar{v}(\vec{s}-\vec{k})}{k^0+w_p+w_{k-s}-i\epsilon}\Big]\gamma^\beta u(-\vec{p}).
\end{aligned} \tag{3.6}
$$

After integrating in k^0 and some simplifications, we obtain

$$
T_{\uparrow\downarrow B} = T_{Bvu} + T_{Buv} \tag{3.7}
$$

where

$$
T_{Bvu} = -\frac{e^4}{2} \int \frac{d^2k}{(2\pi)^2} \frac{w_k+w_p}{\vec{k}^2-\vec{p}^2-i\epsilon} F^*(\vec{k},\vec{p}\ ')F(\vec{k},\vec{p}), \tag{3.8}
$$

$$
T_{Buv} = -\frac{e^4}{2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{w_k+w_p} H(\vec{p}\ ',\vec{k})H^*(\vec{p},\vec{k}), \tag{3.9}
$$

whereas for the crisscross graph it results,

$$
\begin{aligned}
T_{\uparrow\downarrow X} = \frac{e^4}{2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{w_k+w_{k-s}} \Big[&G_1(\vec{k}-\vec{s},-\vec{p},\vec{p}\ ')G_1^*(\vec{k}-\vec{s},-\vec{p}\ ',\vec{p})\\
&+G_2(\vec{k},\vec{p},-\vec{p}\ ')G_2^*(\vec{k},\vec{p}\ ',-\vec{p})\Big].
\end{aligned} \tag{3.10}
$$

In the above formula $\vec{s} = \vec{p} + \vec{p}\ '$ and

$$
\begin{aligned}
F(\vec{k},\vec{p}) &= [\bar{v}(\vec{k})\gamma^\mu v(\vec{p})]\Delta_{\mu\nu}(\vec{k}-\vec{p})[\bar{u}(-\vec{k})\gamma^\nu u(-\vec{p})],\\
H(\vec{p},\vec{k}) &= [\bar{v}(\vec{p})\gamma^\mu u(-\vec{k})]\Delta_{\mu\nu}(\vec{p}-\vec{k})[\bar{u}(-\vec{p})\gamma^\nu v(\vec{k})],\\
G_1(\vec{a},\vec{b},\vec{c}) &= [\bar{u}(\vec{a})\gamma^\mu u(\vec{b})]\Delta_{\mu\nu}(\vec{a}-\vec{b})[\bar{v}(\vec{c})\gamma^\nu u(\vec{b}-\vec{a}-\vec{c})]\\
G_2(\vec{a},\vec{b},\vec{c}) &= [\bar{v}(\vec{a})\gamma^\mu v(\vec{b})]\Delta_{\mu\nu}(\vec{a}-\vec{b})[\bar{u}(\vec{c})\gamma^\nu v(\vec{b}-\vec{a}-\vec{c})].
\end{aligned} \tag{3.11}
$$

To perform the spatial integration, we follow a scheme explained in detail for the $\lambda\phi^4$ scalar theory in [13]. We separate the integration region into two parts through the introduction of an auxiliary cutoff $|\vec{p}| \ll \Lambda \ll m$. In the *low* energy part, $|\vec{k}| \leq \Lambda$, nonrelativistic approximations are done directly in the integrands. In the complementary *high* energy region, $|\vec{k}| \geq \Lambda$, we Taylor expand the integrands around zero external momenta. This procedure is closely related to the methods of effective field theories [14]. With these simplifications, the above expressions become,

$$
\begin{aligned}
T_{Bvu} = & -\frac{4e^4}{\theta^2 m}\int_0^\Lambda \frac{d^2k}{(2\pi)^2}\,\frac{1}{\vec{k}^2-\vec{p}^2-i\epsilon}\frac{\vec{k}\wedge\vec{p}}{(\vec{k}-\vec{p})^2}\frac{\vec{k}\wedge\vec{p}\,'}{(\vec{k}-\vec{p}\,')^2} \\
& -\frac{e^4}{2m^2\theta^2}\int_\Lambda^\infty \frac{d^2k}{(2\pi)^2}\,\frac{(w_k+m)^3}{|\vec{k}|^2 w_k^2}\frac{\vec{k}\wedge\vec{p}}{|\vec{k}|^2}\,\frac{\vec{k}\wedge\vec{p}\,'}{|\vec{k}|^2},
\end{aligned}
\tag{3.12}
$$

$$
T_{Buv} = -\frac{e^4}{16\theta^2 m^7}\int_0^\Lambda \frac{d^2k}{(2\pi)^2}\,(\vec{k}\wedge\vec{p})(\vec{k}\wedge\vec{p}\,') - \frac{e^4}{2\theta^2 m^2}\int_\Lambda^\infty \frac{d^2k}{(2\pi)^2}\,\frac{(\vec{k}\wedge\vec{p})(\vec{k}\wedge\vec{p}\,')}{w_k^2(m+w_k)^3} \tag{3.13}
$$

and

$$
T_{\uparrow\downarrow X} = \frac{e^4}{m\theta^2}\int_0^\Lambda \frac{d^2k}{(2\pi)^2}\frac{(k-p)_-}{(\vec{k}-\vec{p})^2}\,\frac{(k-p')_+}{(\vec{k}-\vec{p}\,')^2} + \frac{e^4}{\theta^2}\int_\Lambda^\infty \frac{d^2k}{(2\pi)^2}\frac{1}{w_k\vec{k}^2}, \tag{3.14}
$$

where $a_- = a^1 - ia^2$ and $a_+ = a^1 + ia^2$. Performing the integrals and keeping only the dominant terms in $1/m$, we find,

$$
T_{Bvu} = \Big[\frac{e^4}{4\pi m\theta^2}\ln\Big(-\frac{q^2}{p^2+i\epsilon}\Big)\Big]_{low} \tag{3.15}
$$

$$
T_{\uparrow\downarrow X} = \Big[\frac{e^4}{4\pi m\theta^2}\ln\frac{\Lambda^2}{q^2}\Big]_{low} + \Big[\frac{e^4}{4\pi m\theta^2}\ln\frac{4m^2}{\Lambda^2}\Big]_{high}. \tag{3.16}
$$

We observe that, up to leading order, T_{Buv} and the *high* part of T_{Bvu} vanish. Differently from what happens in the Galilean formulation [6], our result is finite,

$$
T_{\uparrow\downarrow B} + T_{\uparrow\downarrow X} = \frac{e^4}{4\pi m\theta^2}\ln\Big(-\frac{4m^2}{p^2+i\epsilon}\Big). \tag{3.17}
$$

Nevertheless, it should be noticed that if we consider only the low energy contributions and reinterpret Λ as an ultraviolet cutoff then the scattering amplitude for the crisscross and box diagrams diverges logarithmically,i. e.,

$$[T_{\uparrow\downarrow B}+T_{\uparrow\downarrow X}]_{low}=\frac{e^4}{4\pi m\theta^2}\ln\Big(-\frac{\Lambda^2}{p^2+i\epsilon}\Big). \tag{3.18}$$

This is exactly what happens in the nonrelativistic theory were the amplitude only becomes finite after the addition of a quartic counterterm (Eq. (3.18), up to an overall phase, agrees with the last formula of [6] for $ss'=-1$ and $M=M'=m$, after the identification $g^2=e^2/\theta$). In our formulation however the needed counterterm is automatically provided by the contribution from the *high* energy part of the above integrals. In fact, in the nonrelativistic formulation the last term in (3.16) can be effectively interpreted as coming from the counterterm

$$\frac{e^4}{2\pi m\theta^2}\ln\frac{\Lambda}{2m}(\bar{\varphi}\varphi)(\bar{\psi}\psi), \tag{3.19}$$

which suggests that a $(\bar{\varphi}\varphi)(\bar{\psi}\psi)$ interaction should be considered from the starting in the nonrelativistic model. The extra coupling parameter associated with such interaction could be adjusted to reproduce up to this order the expansion of the non-perturbative AB effect, canceling the one-loop contributions.

The whole scattering one loop amplitude of one spin up and one spin down fermions is given by the sum of Eq. (3.1) and (3.17)

$$T_{\uparrow\downarrow R}+T_{\uparrow\downarrow B}+T_{\uparrow\downarrow X}=\frac{2e^4}{3\pi m\theta^2}+\frac{e^4}{4\pi m\theta^2}\ln\Big(-\frac{4m^2}{p^2+i\epsilon}\Big) \tag{3.20}$$

and it is finite. The corresponding result for the scattering of two spin up fermions was calculated in [5]. Taking into consideration the modification in the vacuum polarization due to the presence of the spin down fermion one obtains the following one loop amplitude for two spin up fermions before anti-symmetrization,

$$T_{\uparrow\uparrow R}+T_{\uparrow\uparrow B}+T_{\uparrow\uparrow X}=\frac{e^4}{6\pi m\theta^2}. \tag{3.21}$$

As remarked in [5], in the case of just one flavor this contribution actually vanishes after anti-symmetrization. However, in Eq. (3.20) the two fermions are distinguishable and no anti-symmetrization is required; it survives as it stands.

The effective low energy Lagrangian emerging from our study is given by

$$
\begin{aligned}
\mathcal{L}_{eff} = {} & \psi^\dagger (i\frac{d}{dt} - eA^0)\psi - \frac{1}{2m}|\vec{\nabla}\psi - ie\vec{A}\psi)|^2 + e^2/2\pi\theta)\, B\psi^\dagger\psi \\
& +\varphi^\dagger (i\frac{d}{dt} - eA^0)\varphi - \frac{1}{2m}|\vec{\nabla}\varphi - ie\vec{A}\varphi|^2 - \frac{e}{2m}\,(1 - e^2/2\pi\theta)\, B\varphi^\dagger\varphi \\
& +\frac{\theta}{4}\epsilon_{\mu\nu\rho}A^\mu F^{\nu\rho} - \frac{1}{4}\left(\frac{e^2}{6\pi m}\right) F^{\mu\nu}F_{\mu\nu} + \frac{e^4}{2\pi m\theta^2}\ln\frac{\Lambda}{2m}(\bar{\varphi}\varphi)(\bar{\psi}\psi).
\end{aligned}
\tag{3.22}
$$

Up to one loop this Lagrangian summarizes the low momentum behavior of a system of two relativistic fermions interacting through a CS field. It differs from the usual Pauli–Schrödinger Lagrangian for nonrelativistic fermions minimally coupled to a CS field, due to the incorporation of purely quantum high energy effects: anomalous magnetic moments, vacuum polarization and of a quartic fermionic counterterm. This last term results from the contribution of relativistic intermediate states to the scattering amplitude of spin up and spin down fermions. The parameter Λ is to be understood as an ultraviolet cutoff to be used in the computations with the above effective Lagrangian.

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[8] We use natural units ($c = \hbar = 1$) and our metric is $g_{00} = -g_{11} = -g_{22} = 1$. The fully antisymmetric tensor $\epsilon^{\mu\nu\lambda}$ is normalized such that $\epsilon^{012} = 1$ and we define $\epsilon^{ij} \equiv \epsilon^{0ij}$. Repeated greek indices sum from 0 to 2, while repeated Latin indices from the middle of the alphabet sum from 1 to 2. For the γ-matrices we adopt the representation $\gamma^0 = \sigma^3, \gamma^1 = i\sigma^1, \gamma^2 = i\sigma^2$, where $\sigma^i, i = 1, 2, 3$, are the Pauli spin matrices. The solutions of the free Dirac equation corresponding to particles of spin up an down are given by

$$u(\vec{p}) \;=\; \left(\frac{w_p + m}{2\,w_p}\right)^{\frac{1}{2}} \begin{bmatrix} 1 \\ \frac{p^2 - ip^1}{w_p + m} \end{bmatrix} \qquad v(\vec{p}) \;=\; \left(\frac{w_p + m}{2\,w_p}\right)^{\frac{1}{2}} \begin{bmatrix} \frac{p^2 + ip^1}{w_p + m} \\ 1 \end{bmatrix} .$$

where $w_p = (m^2 + \vec{p}^2)^{1/2}$ and the normalizations were chosen so that $\bar{u}u = -\bar{v}v = m/w_p$.

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FIGURES

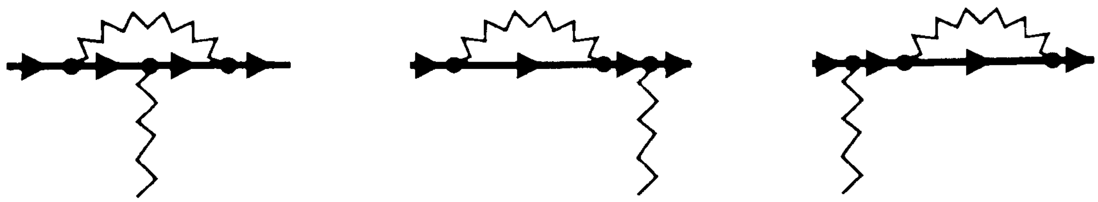

FIG. 1. One loop contributions to the fermion anomalous magnetic moment.

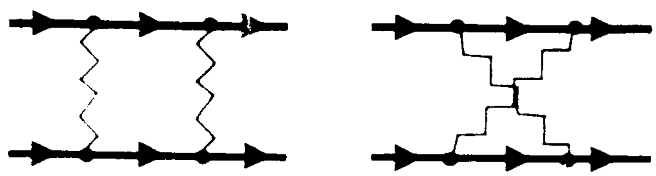

FIG. 2. Graphs contributing to the relativistic fermion-fermion scattering in one loop approximation.